Demystifying "modes" in multi-mode squeezing

Rory W. Speirs*
Joint Quantum Institute, National Institute of Standards and Technology and the University of Maryland, College Park, MD 20742, USA
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Many people are confused about what multi-mode squeezing is. What are squeezed spatial modes? How many are there? What affects how many there are and how 'big' they are? Even some people who think they are not confused probably are, so here is a discussion which should clarify things.

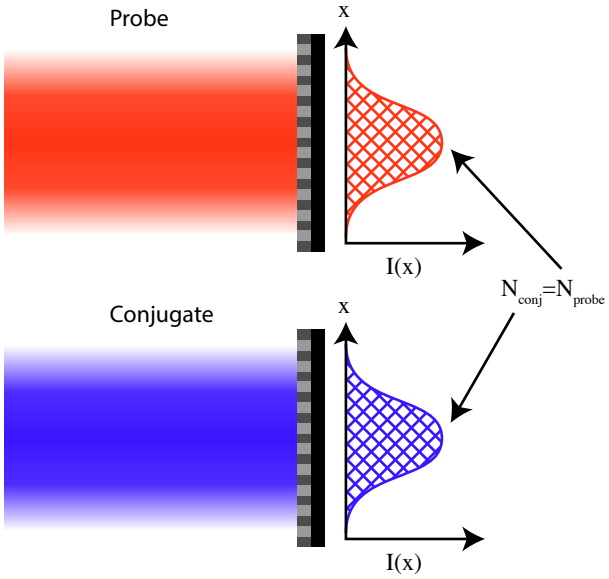

FIG. 1. Two-mode intensity-difference squeezing. The total number of photons falling on the probe and conjugate detectors is the same when integrating over the whole detector.

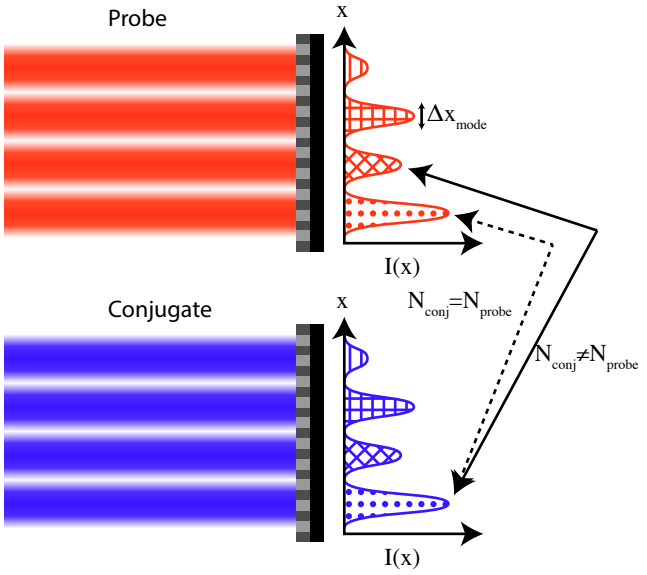

FIG. 2. Multi-mode two-mode intensity-difference squeezing. Comparing the integral over corresponding regions of the detector will yield similar photon numbers, but comparing different regions will not.

I. INTRODUCTION

Before thinking about how we might actually generate multi-mode squeezing, let's just define what we mean by "multi-mode" squeezing in the context of "two-mode" intensity-difference squeezed beams. Firstly, the "two-mode" part signifies that we have two completely separated detectors, one of which is detecting probe photons, and the other is detecting conjugate photons. The probe and conjugate are at different optical frequencies, so there is no ambiguity here that these are different "modes". The "squeezing" between these two modes just means that if you add up all the photons collected by each detector, this number will be very close (closer than you could get for two coherent laser beams). This is illustrated in figure 1. To be more exact, the total number of photons detected, N, will depend on the spatial intensity distribution that you integrate over, $I(x,t)$, and the time that you collect for:

$$N = \frac{1}{\hbar\omega}\int\int I(x,t)\mathrm{d}x\mathrm{d}t. \tag{1}$$

For this discussion, lets just say that we always integrate over the same amount of time, and so the number of photons in a "mode" is just given by the area under the intensity graph, as indicated in figure 1.

Now, if we have "multi-mode" two-mode squeezing, that simply means that on our detectors, there are multiple sets of these correlated regions, where the number of photons in one region of the probe detector is highly correlated with the corresponding region on the conjugate detector, but not necessarily correlated at all with other regions. A cartoon of this multi-mode system is shown in figure 2, where the individual modes are nicely separated, with size Δx_{mode}. Note that photons belonging to a particular mode could land anywhere in the corresponding lobes at the probe and conjugate detectors, so you do need to integrate over the whole mode in order to get a very high level of correlation between the photon number counts.

In practice, each mode will not normally be completely

* speirs@umd.edu

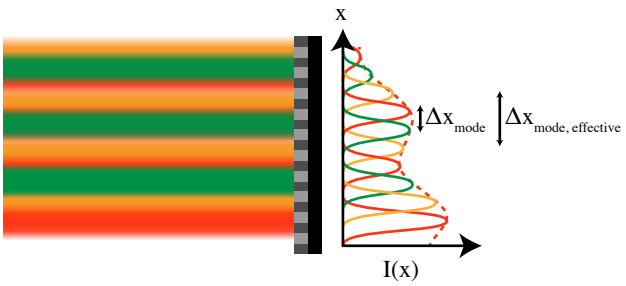

FIG. 3. Realistic multi-mode intensity distribution. A continuous intensity distribution will be made up of a continuum of overlapping modes (each of size Δx_{mode}). The different colours have no significance, they simply help to visually separate adjacent modes.

separated from every other mode, but rather a smoothly varying intensity distribution will be made up of a continuum of closely spaced modes as shown in figure 3.

In this situation, in order to get a very high level of correlation in photon counts when you compare corresponding regions on the probe and conjugate detectors, you will need to integrate over a region that is larger than just a single mode, say $\Delta x_{\text{mode eff.}}$ By integrating over a larger region, you are decreasing the total fraction of your photon count that comes from the edge regions, where modes will have been only partially integrated over. The only thing you can say about a pair of correlated modes on the probe and conjugate detectors, is that if you integrate over the *whole* mode, you will get a very similar photon number. So if you only integrate over some fraction of a mode (which will happen by necessity on the edge of any region that you integrate over), then you can't expect high correlations in the photon number for that mode - hence the need to integrate over a larger area to increase the fraction of whole modes that you include.

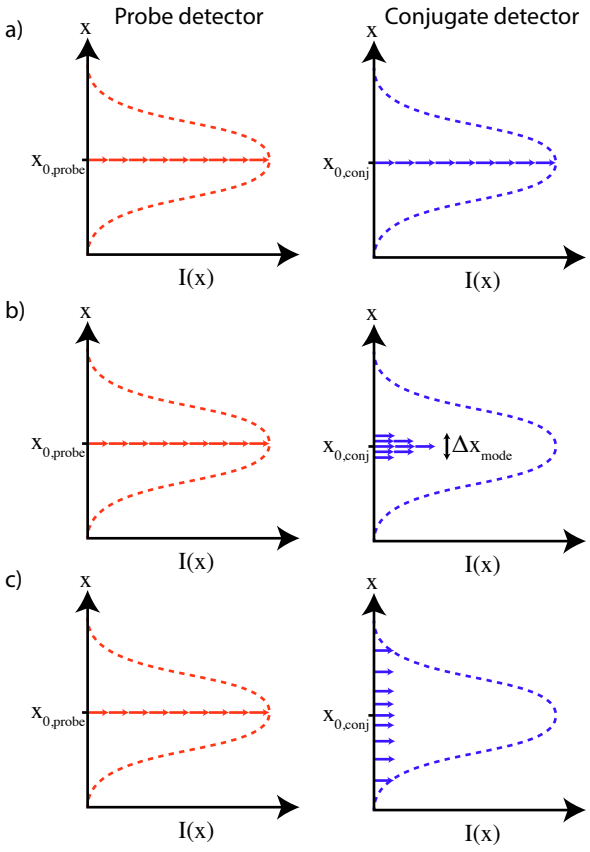

FIG. 4. Clarifying what "modes" are in multi-mode squeezing. Individual probe and conjugate photons are represented by the small red and blue arrows respectively. a) Ideal multi-mode generation. Points on one detector map to points on the other. b) Real multi-mode generation. Points on one detector map to finite sized regions on the other, this defines the "mode size". c) Not multi-mode generation. Probe photons that land at a point on one detector have conjugates that could land anywhere on the other.

II. MULTI-SPATIAL-MODE CORRELATIONS FROM FOUR WAVE MIXING

In a four wave mixing event, an atom will absorb two pump photons, and emit one probe and one conjugate photon. In a regular squeezing experiment, we will collect all probe photons, and put them on one detector, and all conjugate photos will be sent to another (as in figure 1). We don't care about differentiating where the photons fall on each detector.

To have multi-mode squeezing, we need to find a way to map positions on the probe detector to positions on the conjugate detector. This simply means that for any 4WM event, the probe and conjugate photons from *that* event will fall on *corresponding* regions of the two detectors. The other way to say this, is that for any 4WM event, if you detect a probe photon at some position on the probe detector, you can predict where the conjugate photon from that event will fall on the conjugate detector.

To labour the point just a little more, consider the case of a 4WM setup where there is a significant intensity of probe and conjugate photons detected over large regions of the probe and conjugate detectors. Now consider any set of separate 4WM events where the probe photon from each event happens to fall on exactly the same place on the probe detector as shown in figure 4. If all conjugate photons fall on exactly the same place on the conjugate detector, we have the ideal case of infinitely small spatial modes, so the number of different spatial modes will be infinite. If the conjugate photons are actually detected in a small blob centred around a point, we have a finite sized spatial mode, and the number of different spatial modes will be approximately the total size of the illuminated detector, divided by the size of this blob. If the conjugate photons land over the entire illuminated region of the conjugate detector, then you only have a single squeezed mode.

It turns out that with our 4WM process, there are two different ways to send the probe and conjugate photons from any 4WM event to corresponding positions on the

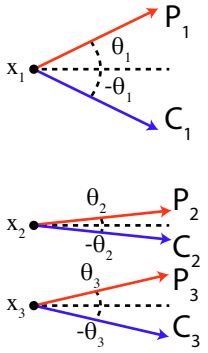

FIG. 5. Positional and angular correlations. For any individual 4WM event, the probe and conjugate photons (P_i C_i) will have both originated from the same location, x_i, and will have equal (but opposite) propagation direction, θ_i $-\theta_\mathrm{i}$. Either correlation can be used to map positions on the probe detector to positions on the conjugate detector.

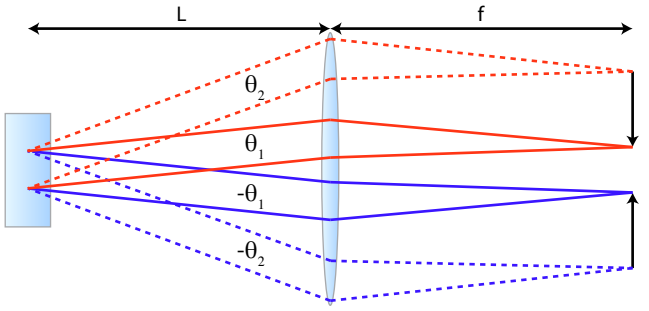

FIG. 6. Mapping detectors together using angular correlations of the probe and conjugate: the "Fourier setup". By placing the detectors one focal length (f) in front of the lens, the lens maps angles of light passing through it to positions on the detectors. The position of the 4WM event in the cell is irrelevant to where the probe and conjugate photons end up.

probe and conjugate detectors: by exploiting either positional, or angular correlations in the generated photons (figure 5).

Angular Correlations

Due to conservation of momentum, the probe and conjugate photons must be emitted at equal and opposite angles relative to the direction that the pump photons were travelling. The 4WM process happens most readily at an angle of $\sim 0.3°$, but still occurs with reduced gain between angles of $0.1°$ and $0.6°$, depending on many experimental parameters. This range of angles can be used to "generate" multiple modes, by finding an optical system which maps different emission angles to different positions on the detector. The optical system which achieves this is simply a lens used in a "Fourier transforming" configuration, where the 4WM atoms are located behind the lens, and the detector is located one focal length in front of the lens, as shown in figure 6. Different emission angles then map to different points on the two detectors, and therefor different points on the two detectors map to each other, as is required to get multi-mode squeezing.

In figure 6 the distance between the atoms and the lens, L is illustrated as being equal to the focal length of the lens. In fact it is not necessary to have $L = f$, as the same mapping from angle to position will result for any value of L at all! What is affected by the choice of L is the phase of the wavefields at the detectors (the Fourier plane). If $L = f$, then the field at the detector will be exactly the Fourier transform of the field at the atoms. If L is greater than or less than f, then the field *amplitude* will be equal to the case of $L = f$, but the phase of the field will now be modulated by that of a spherical wave. This is not important if the field is being detected at this location, as all phase information is lost anyway, but can have consequences if further propagation is required (as in figure 8).

Positional Correlations

Another way to "generate" multiple spatial modes is to recognise that an atom emits a probe and conjugate photon from physically the same region of space. So, if different positions in the cell can be mapped to unique positions on the probe and conjugate detectors, then these position-of-origin correlations can also be used to map positions on the detectors to each other, generating multi-mode squeezing.

Mapping unique positions to unique positions can be achieved with a lens, where both the atoms and detector are placed further than the focal length away form the lens. The exact locations that the atoms and detector must be placed depends on the desired magnification, and can be calculated with the thin lens equation. Exploiting positional correlations is a little bit more difficult than the angular correlations, because you have to find a way to separate the probe and conjugate photons *before* they pass through the imaging lens. If they are not separated, then the lens will map both probe and conjugate photons form a single event to the same region on a single detector, which is not what we want! Note that it will also map any pump photons that pass through the point of the 4WM event to this location, which will swamp any signal produced by 4WM. The solution is to use the relatively large average angular separation between the probe and conjugate to spatially separate these two beams and *then* use an imaging lens, as shown in figure 7.

If it is impractical to separate the beams by propagating them a long distance, then they can be separated using a Fourier transforming lens as in figure 6, but with "pick-off" mirrors placed one focal length if front the lens, instead of detectors. For each beam, a second lens is then placed one focal length in front of the mirror, and the detector is placed one focal length in front of the lens as shown in figure 8.

The atoms in the cell are now being imaged in a "4f" setup, but the use of mirrors in the Fourier plane has allowed separation of the different angular classes that the

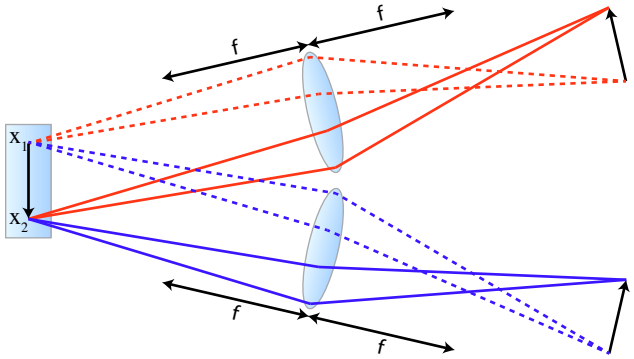

FIG. 7. Mapping detectors together using positional correlations of the probe and conjugate: the "imaging setup". By placing both the atoms and detector further than one focal length from the lens, an image of the field at the position of the atoms is projected onto the detector (the ratio of the distances must satisfy the lens equation to ensure the 'image' is in focus). The positions that the probe and conjugate photons from any one event are detected depend only on where in the cell the atom was when it emitted. Note that this system requires separate lenses, and for the probe and conjugate beams to separate by propagation. In practise, this setup requires long focal length lenses.

probe and conjugate belong to. The actual focal length of the lenses need not be the same, so long as the focus of each lens is at the correct position. Using unequal focal length lenses changes the magnification between the atoms and the detectors.

III. WHAT LIMITS THE NUMBER OF MODES?

Ideally, whichever system we choose to map our two detectors to each other, this map would be perfectly one-to-one, ie. a point on one detector would map exactly to one point on the other detector, and nowhere else. In reality, a point on one detector will map to a small area on the other detector, which is the source of our finite sized mode "lobes" shown in figure 2.

The cause and size of this point spread function in the mapping depends on which system you have used to make the mapping, along with some experimental parameters.

III.1. Mode number from positional correlations

The factors that influence mode size and number are fairly easy to describe for the case of the positional correlation setup. Really, we are just using a lens to produce an image on the detector of the object, which happens to be probe or conjugate light at the location of the 4WM atoms as shown in figure 9. In this case, the size of the mode is actually just the *spatial resolution* of the imaging system, which can be determined using the Rayleigh criterion:

$$\theta_{\text{min}} = 1.22\frac{\lambda}{D}, \tag{2}$$

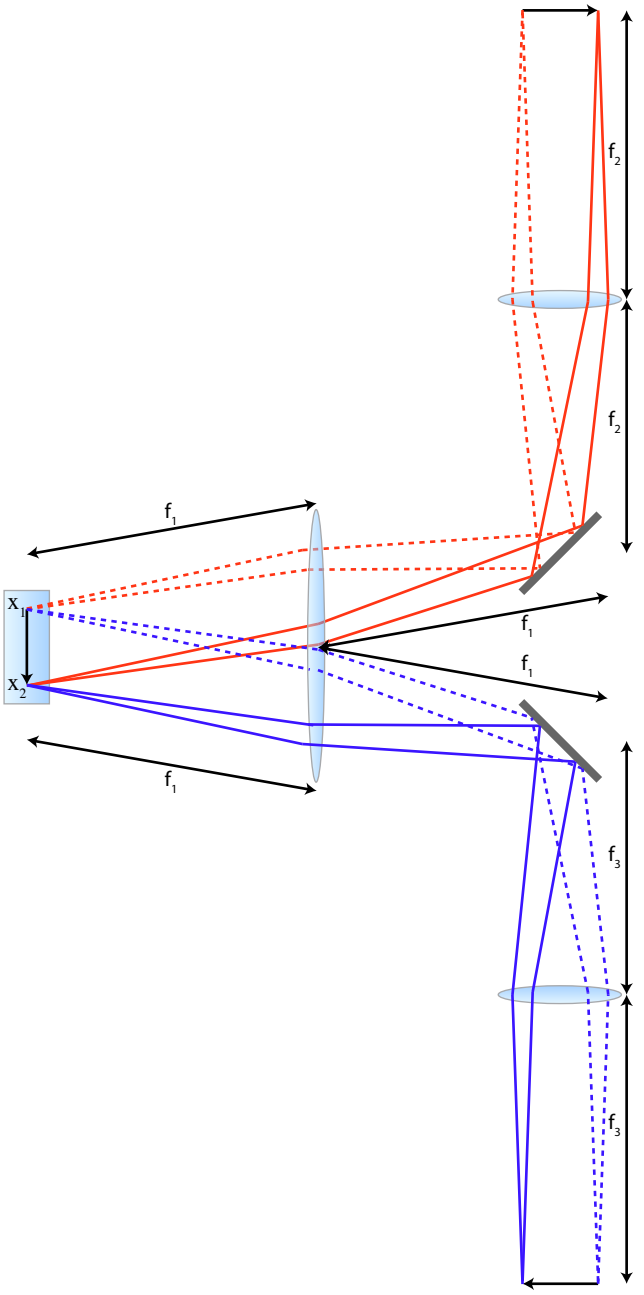

FIG. 8. Using positional correlations to map the detectors together, but a Fourier transforming lens to separate the probe and conjugate light.

where λ is the optical wavelength, D is the diameter of the lens aperture, and θ is the is the minimum *angular* separation of two points located in the object plane. The angular separation of two points spaced apart by a length of Δx which are a distance of L from the lens is just $\theta \approx \Delta x/L$ for small values of θ. Subbing this into the Rayleigh criterion gives the spatial resolution of the system, and hence also the mode size:

$$\Delta x_{\text{mode}} = 1.22\frac{\lambda L}{D} \tag{3}$$

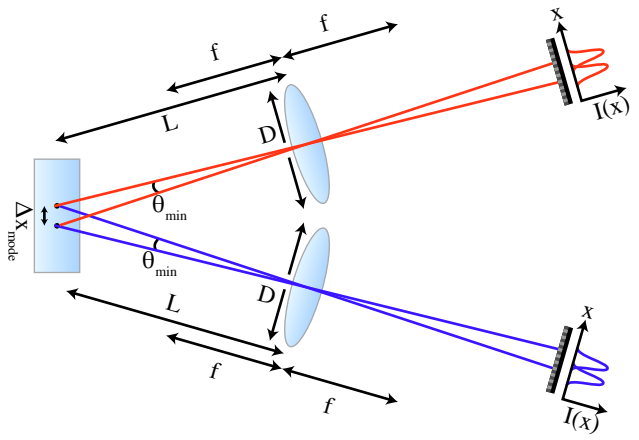

FIG. 9. Mode size in the positional correlation "imaging" layout. Mode size is really just the spatial resolution, which is determined by the Rayleigh criterion.

The total number of modes is just the total size of the region from which you produce any squeezed light, divided by the mode size. Since the light is produced in the region illuminated by the pump, the total number of modes scales with the size of the pump beam. However areas that are not also illuminated by the probe seed beam will not produce many 4WM events, and so these modes will be very dark. For some purposes (such as direct intensity difference imaging) these dark modes may not be very useful, so for practical purposes the size of the overlapping pump and probe beams determines the number of modes.

Up to this point the "mode size" at the detector has not been differentiated to that at the atoms. If the imaging system has a magnification of 1, then these values are indeed the same, otherwise a magnification factor of the imaging system, M, relates the two sizes:

$$\Delta x_{\text{mode(det)}} = M \Delta x_{\text{mode(atoms)}} \tag{4}$$

Spatial resolution: apertures or beam size?

It is perhaps surprising that the image resolution, and hence mode size, is determined by the size of imaging lens, and not the size of the probe/conjugate beam. You might think that since the beam just looks like a regular beam, then it will only go through the centre of the lens anyway, so making the lens bigger cannot possibly make any difference to how much spatial detail can be resolved at the atoms. This is, however, not the case.

Whenever a four wave mixing event occurs, the amplitude of the optical field is increased over a small region of space about that atom. As the optical field propagates, this local increase in amplitude spreads out, because this is just how optical fields propagate. Note that even if only the *amplitude* of the field is changed locally, leaving the *phase* undisturbed, upon propagation both the amplitude and phase of the total wavefield will be modified.

Our situation of adding extra light to the beam is exactly analogous to absorption imaging of an atomic cloud using a laser beam. In absorption imaging, you start with a beam with high amplitude, and every atom that absorbs a photon locally reduces the amplitude of the field. If you happen to be imaging a very homogenous, smoothly varying object, not much light is diffracted out to high angles, but the resolution of your imaging system is set by the size of the lens aperture nonetheless. Perhaps the next shot you take might contain some sharp edges, which will diffract light out further. *The resolution of your imaging system is determined by your imaging system, not by the object you are imaging.*

III.2. Mode number from angular correlations

In the setup using angular correlations to map positions on the detectors together, clearly the modes are not separated by position. Instead, the minimum 'separation' between two modes is given by the smallest difference in emission angle of two photons produced by the atoms that we can reliably map to distinct positions on the detectors. In this case we can again use the Rayleigh criterion, but there are some important differences in how we must apply it.

In this case, it turns out that the roles of the pump beam size (or possibly pump/probe overlap) and the lens aperture are reversed. The number of modes that can possibly be collected is determined by the size of the lens aperture, whereas the 'size' of the mode (the minimum difference in angle that can be resolved), is determined by the size of the pump beam (or possibly pump/probe overlap)! This is on the condition that the lens aperture is larger than the pump/probe beams at the position of the lens.

Let's first discuss the minimum resolvable angular difference. Ultimately whether or not two different angles can be called separate modes depends on if they are spatially separated on the detector. In the Fourier transforming setup, the spot size formed for a given angle of input beam is determined by the physical side of that beam as it passes through the lens. Ie, in the Fourier transforming setup, the "aperture" in the Rayleigh criterion, is actually the size of the probe beam D_{seed} (possibly the pump, but I really don't think so).

$$\Delta\theta_{\text{mode}} = 1.22 \frac{\lambda}{D_{\text{seed}}}, \tag{5}$$

Now, the number of modes is determined by the size of the lens aperture only because this limits the total range of angles that can be accepted. For a given aperture size, if the lens is closer to the cell, it will accept greater angles and hence more modes, as seen in figure 10. However very large angles will not contain much light because the 4WM process has very low gain at high angles, so these modes will be dark. In practise then, the number of modes scales with the angular spread of the probe beam. The angular spread increases as the size of the beam focus inside the

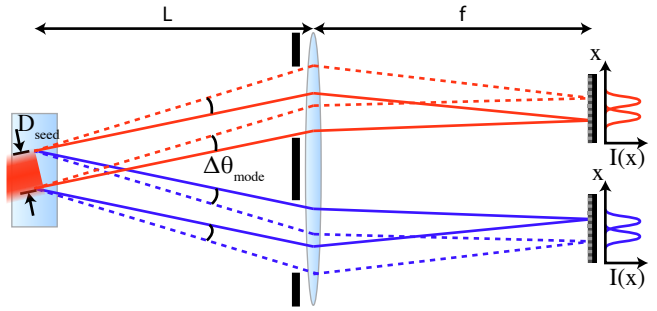

FIG. 10. Mode number and size in the Fourier setup. Mode size is determined by the probe seed beam size, as larger beams can focus more tightly. Mode number is limited by lens aperture, as this determines the maximum angle that will be admitted through the lens. From a practical standpoint, the mode number is actually limited by the range of the angles for which 4WM occurs, which itself is limited by the range of seed angles present and the 4WM phase matching conditions.

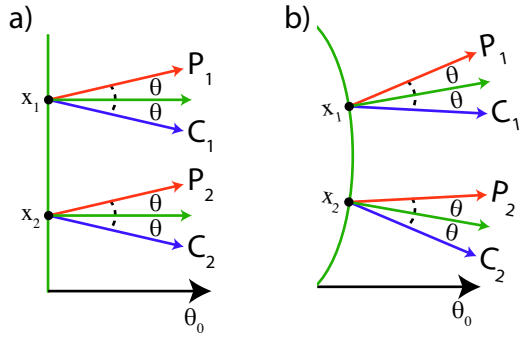

FIG. 11. Pump wavefront curvature and "mode size". Pump wavefront curvature increases effective mode size on the detectors when using the angular correlations to create a mapping between the two detectors. a) Flat pump wavefront. Two events with the same angle relative to the local pump angle are parallel to each other, so get focused to the same position. b) Curved pump wavefront. Two events with the same angle relative to the local pump angle are no longer parallel to each other, so are focused to different positions. This increases $\Delta\theta_{\text{mode}}$.

cell decreases, so a very small probe beam will give a large number of modes in this setup, which is opposite to the imaging setup!

The final thing to consider with this setup is the wavefront flatness of the pump. If the pump were a perfect plane wave, then the angular mode size would be just given by equation 5. However, if the pump has some wavefront curvature, then two 4WM events that occur in different locations but with the same emission angle relative to the pump photon direction, will not have the same angle relative to each other, as illustrated in figure 11. This will have the effect of creating different mappings between the two detectors, depending on where in the pump beam the 4WM event occurred. Ultimately this just blurs the mapping, effectively resulting in spatially larger mode sizes at the detectors. It happens that the larger a beam is, the more flat you can make the wavefront. And this is the origin of the oft repeated saying "the mode size is determined by the size of the pump beam". But as we have seen, the factors that determine "mode size" depend what kind of correlations between the probe and conjugate photons you are using to map the two detectors together.

IV. WHY CAVITIES RESULT IN A SINGLE SQUEEZED MODE

It is worth briefly mentioning why the addition of cavities around the atoms for the probe and conjugate beams reduces the number of squeezed spatial modes to only one. In a cavity with only a single occupied spatial mode, light generated at one position of gain medium bounces back and forth in the cavity a number or times, so the local increase in field amplitude spreads out across the entire wavefront. At this point, the light generated from that particular 4WM event is indistinguishable from light generated by a 4WM event at a different position, so when the light finally does exit the cavity, it has the same likelihood of being detected at a particular location on the detector as any other 4WM event. If the cavity has multiple spatial modes occupied, then it is totally possible that 4WM events with a particular emission angle or location will contribute to one cavity mode, and other locations or angles to another cavity mode. In this case, you could again get multi-mode squeezing, and the different spatial "modes" would actually correspond to cavity modes like Hermite-Gaussian modes. These modes would then have to be separated somehow in order to deposit their light on spatially separated regions on a detector.

V. TWO BEAM COUPLING WITH MULTI-SPATIAL-MODES

Meng-Chang's experiments have shown us that with the dual seeding experiments, you do not want the seeds to cross inside the cell because you end up with extra noise on the squeezing spectrum. The explanation for this is "two-beam coupling", where whenever two beams cross in a nonlinear medium, you can cake photons out of one beam and put them in the other, and vice versa. The exchange of photons between the beams happens stochastically, adding noise to both beams. Note that this only really happens to a large extent at the probe frequency, as the conjugate frequency is further from the atomic resonance.

Two beam coupling has consequences for which setup we want to use to generate multi-mode squeezing. Every time two probe beams cross they exchange photons at random, so we must ensure that any crossing probe beams are detected together, because we know that the total number of photons in the two beams is conserved. Clearly this is only true when using the "imaging" setup which uses the positional correlations of the probe and

conjugate, as shown in figure 12.

It can be seen that in the setup using the angular correlations, probe beams cross each other that are detected at different locations, so the photon numbers here will not correlate as strongly to the corresponding regions on the conjugate detector.

VI. THE GENERAL CASE OF MAPPING THE DETECTORS TOGETHER

We will now briefly generalise our two different methods of mapping the probe and conjugate detectors together. Each 4WM event produces photons that have the same position initially, and opposite momentum (angle) in the x direction. So the probe and conjugate beams can be represented on a phase space diagram as two lobes at the same position, but symmetrical in the position axis as shown in figure 13.

Different methods of mapping any probe and conjugate photons to corresponding positions on the detector are represented by any lines that are symmetrical about the position axis, and do not cross. These lines can be thought of as representing pixels on the detector, and any point they pass through is sent to that pixel. The two mapping systems discussed in this paper using imaging or Fourier setups are represented as straight lines parallel to the momentum or position axes respectively. It is also conceivable to come up with some kind of optical system where the mapping lines are not straight, or of uniform thickness (in fact they need not be "lines" at all). The factors that determine things like total mode number would then be different to either of the two systems presented here.

VI.1. A more concise explanation of spatial modes...

This hasn't been worked into the text smoothly, but it should be said.

A 'mode' is just light that is distinguishable from other light. In our setup, light is generated at a specific location with a specific direction. In principle it is possible to map every position in phase space to a different pixel, so that the total number of spatial modes is effectively just the number of photons that are generated in total (since no two separate photons will ever originate from *exactly* the same position with *exacty* the same angle). In practise any mapping system will have some finite resolution. But fundamentally, the question "how many modes are there?" is equivalent to "how many *distinguishable* photons were generated?".

Now, in the context of squeezed spatial modes, we need to ensure that the probe and conjugate are *mutually* distinguishable. That is, the probe and conjugate from a particular 4WM event must be detected on corresponding pixels of different detectors, and the mapping to the detectors must be symmetric about the x axis, as shown in figure 13.

VII. CONCLUSION

Hopefully this document has clarified the meaning of "multi-mode" in multi-mode squeezing. "Modes" in our system are not some spooky Lagurre-Gauss-Hermite thing that exists in the quantum vacuum, and which is brought into being by the existence of the pump beam. "Mode" is just a word which means that we can differentiate between certain 4WM events, and use the detection of one photon at a certain location to say where we will detect its partner photon.

All of this work has come from me just thinking about the problem, and determining what must be going on. There will be other ways to represent many of the concepts, but I think what I have presented here is the most helpful way to think about the topic, particularly if you are new to it. There are some parts where I am genuinely not sure if I am correct. In particular, I am a bit unsure about the factors that affect the mode size, and total mode number for the imaging or Fourier setups. I think the conclusions I have drawn are mostly correct, but I could be reasoned out of them.

Homework for you: think of a better term for our kind of system than "multi-mode-two-mode" squeezing.

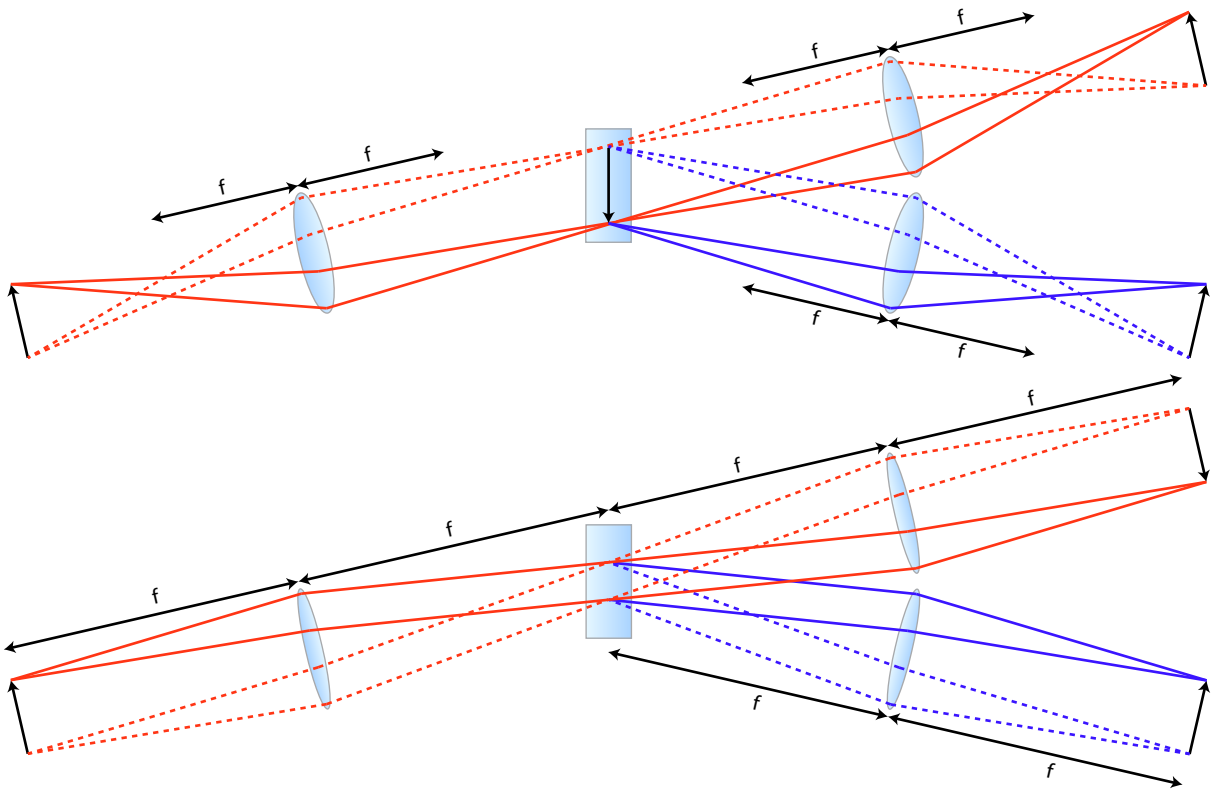

FIG. 12. Position separated (top), and angular separated (bottom) setups. Depending on the optical setup after the cell (to the right), unique four-wave mixing events can be mapped to positions on the detectors by using either the position correlations of probe and conjugate photons (top), or their angular correlations (bottom). Coupling occurs wherever red (probe) beams cross. If crossed beams are detected in the same location (top), the coupling doesn't matter, because the sum of the photons in the beams is constant. If beams that cross are detected in different locations (bottom), then there will be increased noise, because the beams have randomly exchanged photons. Note that in the angular separation setup, we are actually using two separate lenses, rather than the one that was shown in figure 6. This is shown here to allow a more direct comparison between the imaging and Fourier setups.

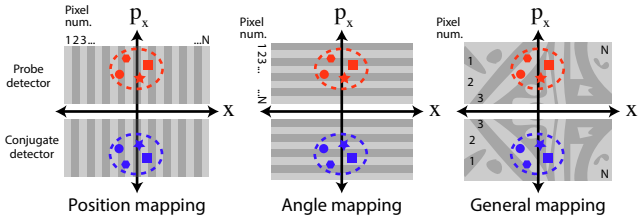

FIG. 13. Detector mappings in phase space. Probe and conjugate photons are generated symmetrically around the position axis. All photons that are cut by a line representing a pixel are sent to that pixel. On the left, the positional correlation setup is represented. The centre shows the angular correlation setup . All possible mappings can work on the condition that they are one-to-one (the lines don't cross), and that they are symmetrical about the x axis. A general mapping example is shown on the right.